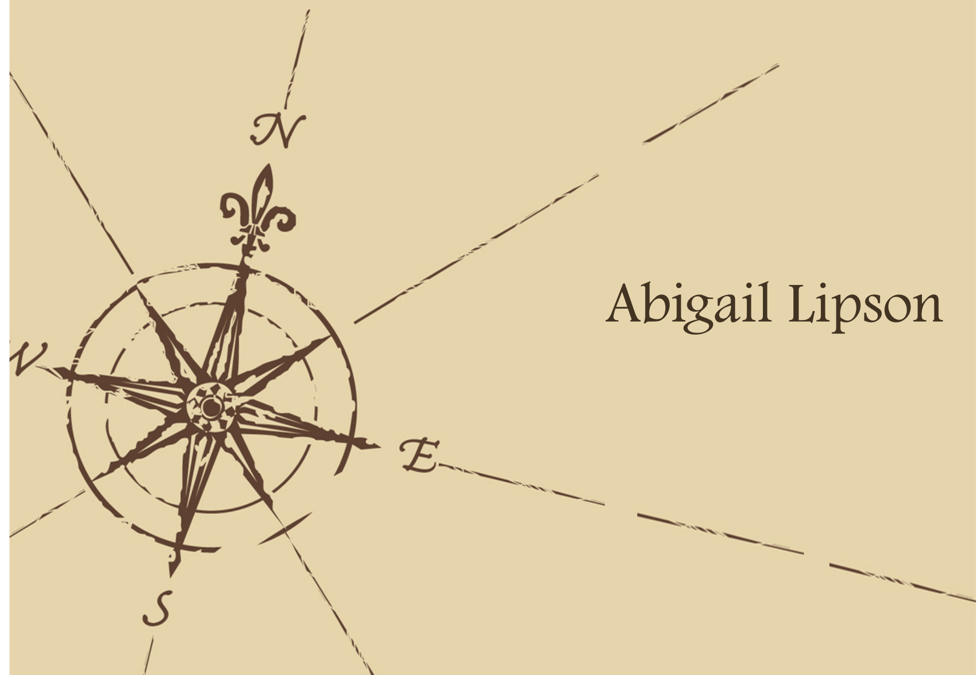
THE FOUR-POINT APPROACH

TO PROBLEM SOLVING IN MATH AND SCIENCE



Abigail Lipson

IN THE COLLEGE YEARS
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IN THE COLLEGE YEARS

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INTRODUCTION

Problems are problematic. It is in their nature to be difficult and time-consuming. But difficulty isn't only a property of the problem itself; it is also a function of how you, the problem solver, approach the problem. A difficult problem will be even *more* difficult if you approach it ineffectively, while the same problem will be considerably less difficult once you start using a few good problem-solving techniques. Furthermore, as you get better at problem solving you will find that difficulty itself isn't such an odious thing; a problem can be fun or fascinating only when it is reasonably difficult.

Problems worthy
of attack
Prove their worth
by hitting back!

Piet Hein

The Four-Point Approach will help you make the most of your time and effort, especially when you are up against tough problems:

POINT 1. BREAK THE PROBLEM DOWN.

POINT 2. WORK FOR AN UNDERSTANDING, NOT JUST AN ANSWER.

POINT 3. GIVE THE PROBLEM TIME.

POINT 4. DO YOUR SUMS! SUMMARIZE WHAT YOU HAVE LEARNED.

This booklet explains *why* each point is important and *how* you can apply the point to your own studies.

It may feel awkward at first, but if you bravely experiment with different ways of incorporating the four points into your current problem-solving style, you will find that even difficult problems will yield to your efforts.

POINT 1.
BREAK THE PROBLEM DOWN

WHY BREAK THE PROBLEM DOWN?

Many inexperienced problem solvers become discouraged at the first sign of difficulty, because they expect their homework problems to have obvious and speedy solutions. After charging headlong into a problem, they quit when they get lost or reach an impasse. They approach the problem looking for a clear path to the solution and give up when they encounter stumbling blocks.

Most problems at the college level simply cannot be solved in one sure-footed sprint to the finish line. This is because most problems (a) involve multiple non-obvious substeps, (b) require that you make use of very freshly-acquired information or skills, and (c) are unlike any problems you have encountered before.

Effective problem solving involves not a quick race to an immediately visible solution, but a persistent meandering through the problem's complexities to arrive at an eventual understanding. A good way to conduct your exploration of a problem is by doing what experienced problem solvers call *breaking the problem down* — or, as others call it, *setting the problem up.*

> *Don't give up if you don't see the solution immediately. The only problems worth doing are those you can't do all at once.*
>
> *David Layzer*

Breaking the problem down accomplishes two things. First, it helps you gain an in-depth understanding of the problem. Second, it helps you subdivide the problem into manageable steps or tasks. After all, once you have figured out the nature of the problem and its component subtasks, you are well on your way toward the solution.

HOW TO BREAK THE PROBLEM DOWN

Below are a few of the things you can do to break a problem down, especially when you don't otherwise know how to begin or how to proceed with the problem.

<u>Parse the problem</u>. Identify and make a descriptive list of the *givens* in the problem. Identify and make a descriptive list of the *un-*

knowns in the problem. Identify and make a descriptive list of the *operations, formulas, or procedures* specified by the problem. Wherever you can, write down how these elements relate to one another.

> *Good problem solvers have learned that analyzing complex problems and ideas consists of breaking the ideas into smaller steps. They learned to attack a problem by starting at a point where they can make some sense of it, and then proceeding from there.*
>
> *Arthur Whimby and Jack Lochhead,*

Trying to parse the problem ensures that you read the problem carefully and thoroughly. It also makes evident any gaps or uncertainties in your problem-related knowledge, so that you can go pursue these by consulting with your course materials, bringing your questions to office hours, or signing up for a peer tutor.

<u>Generate relevancies</u>. For each item on your lists, brainstorm and note down as much information relevant to that item as you can generate. Be sure to spell out exactly how each of these items is relevant to the problem. Generating relevancies helps you prime your brain to think about the problem by activating your knowledge base regarding the problem's elements and the possible relationships between these elements. Some of these relevancies may contribute directly to solving the problem at hand, and some will not—but you may not be able to determine which will and which won't until you generate them and look them over.

> *A common misconception is that, if you're good, math should be done instantaneously or at least very fast. Many students do not realize that mathematicians frequently spend days or even years puzzling over a problem and that finding a solution requires lots of trials and plenty of errors. The only way they have seen it done is in class by a teacher who has thought about the material beforehand and therefore knows all of the answers right off.*
>
> *Deborah Hughes Hallett*

<u>Locate models</u>. Can you recall, hunt down, or invent models for this problem? Exactly what is this problem an example of? What is its fundamental structure? How are the elements of this problem similar or dissimilar to other problems you have seen?

Locating models will help you understand the problem and identify possible routes to a solution. This is not a suggestion to "plug-and chug" – to locate an exactly analogous problem and solve this one in exactly the same way simply by plugging in different values. The plug-and-chug method tends to be a poor problem-solving method for two reasons: (a) it provides you with only a very superficial understanding of the problem, and (b) it often results in wrong answers, since superficially analogous problems can be entirely different in their fundamentals.

Represent the problem schematically. Draw a diagram. Make an outline of the problem statement. Create a chart or table of relevant functions. One of the best ways to represent a problem schematically, especially a physics problem, is to make a mental "movie" of the problem in your head. Try to imagine the problem elements in time and space, and watch what happens as the problem unfolds.

Representing a problem schematically helps you get a concrete and visual sense of the problem as a whole, along with a sense of how the problem elements relate to one another.

Articulate what you don't know. If you find the problem confusing, try to write down exactly what it is that is confusing you. Write down what sort of information you feel you might be lacking, what operations you aren't clear about, what relationships you don't understand. Articulating what you don't know allows you to systematically go back and fill in gaps in your knowledge or experience.

POINT 2.
WORK FOR AN UNDERSTANDING, NOT JUST AN ANSWER

WHY WORK FOR AN UNDERSTANDING?

Your immediate task may be to complete your current problem set. But you have a larger task; to develop your ability to think and work productively in the subject area. Problem sets are a means to this end rather than an end in themselves. Solving problems helps you (a) consolidate your *knowledge* of the relevant facts or concepts at work and (b) learn *how* to solve problems, thus increasing your ability to solve other problems.

Your main goal in any case is to get the problem set *right*, right? Well, yes. But most students recognize that it is often possible, by means of guesswork or mimicry, to generate correct answers without really satisfying either of the two important purposes described above. That is, it is often possible to get a problem "right" without really understanding the concepts involved and without really improving your ability to solve other problems in the future.

Students who work *just* for answers eventually start feeling frustrated and unsatisfied. They put in a lot of drudge work but don't seem to gain anything substantial from their efforts.

Although working just for answers doesn't necessarily produce much depth of understanding, working for understanding does tend to produce better problem-solving outcomes. So even though working for understanding sometimes feels slower and more difficult than working just for answers, the payoff is immeasurably greater.

HOW TO WORK FOR UNDERSTANDING

Try stuff. Almost anything you actively do or try, regardless of whether or not it is immediately successful, will be of greater benefit to your understanding than passively waiting for a solution to hit you like a bolt from the blue, or expecting concepts to jump from the page into your head of their own volition.

When you are introduced to a new formula, for example, try stuff with it. Try making a chart of what happens when you incrementally change the value of one of the variables. Try putting very large or very small values into the equation and see what happens. Try to invent two situations as different from one another as possible, in which the formula might apply. Try to make up stuff to try! Similarly, when you get a new problem, play with it, manipulate its elements, try stuff with it. Trying stuff will help you achieve a deep understanding, rather than a superficial one.

> *"Let's try it and see what will happen" solves many a problem. It is one of the main streets of the scientific approach.*
>
> *Edward Hodnett*

Document your thoughts. Despite the myth that good problem solvers just think for a second, figure everything out in their heads, and then write out the complete solution without a pause, experienced problem solvers actually tend to use thinking aids (like paper and pencil) *more*, not *less*, than beginning problem solvers. So when you try stuff, try it on paper. This way you can keep track of what you are thinking, how it goes right and how it goes wrong, where your thinking is clear and where it is fuzzy.

Documenting your thoughts serves another purpose as well. Often, there are just too many things to keep in mind all at once while you're working on a problem, and your mental energies are fully occupied just trying to remember them all. Keeping track of some of this information with pencil and paper —"downloading" it onto paper — serves to free up some of that mental energy so that you can devote your attention to the work of problem solving.

Pay special attention to errors. Errors are an important source of information in problem solving — even more so than correct answers. You can examine your errors *while* you are working by stopping every so often to take stock of how you are doing. Or you can examine your errors *after* you have completed your work, by reviewing your corrected problem sets or exams. Did you make a calculation error? Do you make lots of them? Did you misunderstand a concept? In what way, exactly, did you misunderstand? Were you working on the basis of some faulty assumption? What was it and how can you correct it?

Getting it wrong is part of getting it right.

Charles Handy

In the short run, attending to your errors helps you catch them in time to correct them, and helps ensure that you don't make the same mistake twice (or five times). In the long run, attending to your errors contributes to your capacity to think clearly through a problem, errors and all.

POINT 3.
GIVE THE PROBLEM TIME

WHY GIVE THE PROBLEM TIME?

You have no doubt noticed that Point 1 and Point 2 take time. *Lots* of time. College students often underestimate the time that problem solving really takes. Have you ever found yourself thinking…

> *This is taking more time than it should! It didn't used to take so much time in high school. If I were any good at this, it wouldn't take so much time.*
>
> *Why can't I see the answer right away? If I don't see the answer, something must be wrong. I mean, it's really not fair to ask such impossible questions.*
>
> *I don't have time for this! The problem set is due tomorrow! I need the answers NOW!*

The truth is, problem solving is a process that *takes time*. Problems take time because they require that you put your factual and procedural knowledge to use. They take time because they typically require several steps to solve and the solution can be far away and around a corner from your starting point. Problems are different in this way from simple factual questions, to which you either know the answer or you don't, as soon as you read the question.

Giving the problem time allows you to explore the problem thoroughly, which is an essential aspect of good problem solving. (Remember Point 1, above, about breaking the problem down.) More importantly, giving the problem time allows you to extend your "courage span" to *include* the inevitable false starts, confusions, and dead ends that problem-solving requires, rather than interpreting these experiences as a sign that you have already failed and might as well quit.

One must learn by doing the thing, for though you think you know you cannot know for certain until you try.

Sophocles

HOW TO GIVE THE PROBLEM TIME

Monitor your time. Keep track of how long your problem sets actually take you. Base your study schedule on this realistic assessment rather than on your assumptions about how long you *think* the problems should take or how long you *wish* they took or how long they take anyone *else*.

Incorporate your problem sets into your studying. Problem solving and fact learning are both improved when they are done in tandem, reinforcing one another. If you diligently read your textbook but wait until the last minute to lay eyes on your problem set, you will have lost the benefit of this mutual reinforcement. Try instead to get a running conversation going between your readings, problems, and lectures. Formulate questions in one of these contexts and appeal to the other contexts for answers.

> *I would like to introduce my notion of the "courage span" as a possible device for helping students better grasp their latent problem-solving powers. It is a term I derive by analogy from the familiar "attention span." Very simply, the courage span is the time which elapses between the taking on of a problem and the abandonment of that problem.*
>
> *Richard Wertime*

For example, make a guess, just from looking over the problems when you first get them, as to what concepts or operations are most central to the subject matter at hand. Then confirm or revise these impressions as you engage with your readings and lectures. Similarly, when you learn new concepts in class, invent some problems that put the concepts to use — after all, this is what your teachers will have done to design your problem set for that week. Compare the problems you invented yourself to the problems in your problem set.

This sort of speculative thinking is a very useful preparatory step in solving complex problems and, like many study habits, it gets faster and easier the more you practice it.

Take the time to just *work* on a problem without rushing to *solve* it. For example, write down a list of the concepts or procedures you will have to learn in order to solve the problem, or write down questions that the problem brings to your mind. The better part of effective problem *solving* is actually extended problem *working*. In particular, take the time to understand the question. What exactly are you being asked to do or find? One of the most common causes of confusion in problem solving is a failure to read the problem carefully.

Don't treat your problem set as though it is a one-shot time-limited test. A problem set is an exercise, not an exam: an exercise to puzzle through, fiddle with, and ponder. Its purpose is to help you learn, not just to assess what you already know. So you can do the problem set with an open book, or even talk it over with other students, as long as this sort of collaboration is permitted by your instructor.

When you get stuck, try to articulate what is making you stuck, what it is that you don't understand, and then read some more and try it again, or go to your teacher for help, or sign up for a tutor. You have lots of shots at the problem; don't restrict yourself artificially to just one, by leaving your assignments until the last minute. You will have plenty of time-limited closed-book tests; don't turn your problem set into another one.

POINT 4. DO YOUR SUMS! SUMMARIZE WHAT YOU HAVE LEARNED

WHY SUMMARIZE WHAT YOU HAVE LEARNED?

Many students complain that they can never tell what they know and what they don't know. You may *think* you know a concept or formula because you recognize it on the blackboard or because it looks familiar when you come across it in a book. But then you find you can't remember it when you most need it (like when you are sitting at home working on a problem or sitting in class working on an exam). Similarly, you may *think* you can do problems in a given area because you have turned in a correct problem set, only to find you don't do at all well on new problems on the very same topic. Luckily, this sort of academic amnesia can be cured!

The difficulty rests with a mistaken assumption about how people gain knowledge. Students often assume that if you understand something you hear or something you read, then you've "got it." Or if you have completed a problem correctly once, then you "can do that," you've "had that already."

In fact, the knowledge you have gained through your studying may only be understanding on the level of *recognition*. That is, when you see something similar you'll recognize it as familiar. In contrast, most of the knowledge you'll actually need in order to solve more problems or answer exam questions is not understanding on the level of *recognition* but rather understanding on the level of *generation*. That is, your knowledge will need to be available to you in such a form that you can generate it all by yourself out of "thin air." The very best way to ensure that your knowledge will be available to you in this form is to study in ways that give you practice at *generating* your knowledge rather than simply *recognizing* it. And one of the very best ways to do this is to regularly *summarize what you are learning in your own words*.

> *Only by trying new avenues of thought, putting down first one idea and then another, turning a diagram over and over in your mind, doing calculations or checking procedures, can you learn and solve problems. ...The essence of doing math is to keep going.*
>
> *Sheila Tobias*

HOW TO SUMMARIZE WHAT YOU HAVE LEARNED

Close the book. When you get the feeling that you "know" something — a formula or a concept or whatever—test yourself by closing your book and generating your understanding of the material in your own words. This does not mean closing the book and quickly parroting the last few words you have read. It means closing the book, collecting your thoughts, and writing down in your own words your understanding of what you have just learned.

You may be surprised by how difficult this can be, even with something that just a second ago you thought you understood quite well. It may take several tries to master even a small amount of material. But only this sort of practice exercises your ability to generate, not just recognize, your new knowledge.

The moral of the story is... You can use the same method to promote your fundamental understanding of complex concepts. For example, when you have finished working out a problem set, put it aside and see if you can generate a summary of the problems and their solutions, in writing. It doesn't matter if you get every detail right; your effort should be to capture in your own words the main point, the major themes of the problems, and the main lessons learned in their solution. Your goal isn't to test your memory of these particular problems, but rather to try to articulate your understanding of the problems' purpose and structure and to draw a moral, so to speak, of what solving the problems has taught you.

Talk about it out loud. Ask a friend or classmate to be your study-buddy for an hour or so and try the following experiment. Set a timer to go off in ten minutes. Each of you study your own material in silence (reading a part of a textbook, breaking down a problem in your problem set, whatever—the two of you needn't even be studying the same thing). Study with the understanding that when the timer goes off, you will each take two or three minutes to summarize for the other what you have learned and what you feel you don't yet understand. Then when the timer goes off, and you have traded explanations, then set the timer again and repeat the process.

You will reap many benefits from this exercise. First, just knowing you are going to be summing up the material helps you work purposefully and maintain your concentration. Second, summarizing aloud what you have learned is a powerful aid to long-term memory and promotes your capacity to generate, not just recognize, your understandings. Third, articulating to your companion what it is that you don't yet understand or aren't yet clear about helps you refocus your efforts productively during the next ten-minute period. This exercise is equally helpful when you are studying a text or when you are working on a problem set.

> *...A sudden "flash of insight" into a problem is not enough. It is necessary to go on and to formulate the solution precisely in words or symbols.*
>
> *Henry Maddox*

Hit the "memory hot spot." One thing that makes summarizing so powerful is that it hits the "memory hot spot." The hot spot is the period of time *immediately* after you gain new knowledge or acquire new information. It is during this critical period that you can either lock your new knowledge into your memory or let it float away into oblivion.

When someone gives you their new phone number and you want to learn it so that you'll be able to generate it from memory whenever you need it, what do you do? Probably, you *immediately* repeat it to yourself several times, you say it out loud, you pause for a moment and then try generating it again. This is an extremely successful and effective tactic, and you can apply exactly the same tactic to your studying and problem solving.

There are many ways to make good use of the memory hot spot in your problem solving. One of the most powerful ways is related to your classroom learning. The memory hot spot with regard to classroom lecture material is the ten minute period immediately after a class ends. If you are like most students, you completely miss the benefit of this hot spot because as soon as the class is over you stand up and rush out the door, or jump into a casual conversation with a friend, or simply turn your attention to the next thing on your schedule. So the hot spot goes unused, and within a half an hour most of what you just learned is gone from your memory. It is *long* gone by the time you get around to working on the problem set.

Two minutes — even ninety seconds — is enough time to get tremendous benefit from the post-class hot spot. Stay in your chair with a paper and pencil, or mumble under your breath while you walk out of the room, or rally the cooperation of a friend in the class, and take these few moments to summarize (or at least roughly list out) in your own words the central ideas or concepts or information you learned in the class. Remember, this is a hot spot in which even a small or messy effort on your part will give a comparatively great boost to your memory and understanding.

STUDENT VOICES

In preparing this booklet, we asked a number of students who had done well in their introductory science courses to tell us how they did it — what was their approach to problem solving, and what words of wisdom would they like to impart to their fellow students. These students said that, in addition to the four points already discussed here, they would offer the following advice:

> *If you are having trouble understanding something, don't assume that it's your fault or that you're stupid. It may be that the material itself is unclear or badly worded. Go to another student or a teaching fellow and ask them to help you figure it out.*
>
> *Always check your answer against the question! Are you answering what the question asks? Is your answer in the right units? If the question asks for a temperature and your answer is a weight, you have gone wrong somewhere!*
>
> *Don't get intimidated by the numbers and formulas to such an extent that they lose their meaning. Suppose your calculations result in an answer of 50 kilograms. That's a perfectly good number. But if the question was asking about the weight of an atom, something in you should say, "Whoa! That's one big atom!" A lot of errors are made when you lose track of the meaning.*
>
> *There is no single right way to study, and no perfect approach to problem solving. You have to experiment to find out what works for you.*
>
> *Go for partial credit on problem sets and exams! Do whatever parts of a problem you can do, and then write down what is making it impossible for you to do the rest. For example, "I know that* ***a*** *varies as a function of* ***b****, but I just can't see how* ***a*** *and* ***c*** *relate!" or "I have a hunch that this next step has something to do with* ***x*** *because of the formula* ***z****, but what is stumping me is how to account for* ***y****? Here's what would happen if we don't take* ***y*** *into account…" Sometimes, the process of writing down what is stumping you actually helps you solve the problem. Even if it doesn't, you may get partial credit for what you do put down.*
>
> *Many approaches that work well in science and math courses also work well in non-science courses and vice versa. For example, the memory hot spot is just as hot after a literature lecture as after a chemistry lecture. If you tend to do well in any particular area, try to adapt what you do in that area to other areas.*

DARING, CURIOSITY, AND PERSISTENCE

Incorporating the Four-Point Approach into your study habits translates directly, sometimes immediately, into improved problem-solving performance. This is good to know, but it may still be daunting to imagine actually implementing the approach. Who has the discipline to remember the four points, to follow them, and to keep on following them? Who has the *time*? And there are so many specific suggestions here, how can anyone take them all?

But really effective problem solving is less a function of discipline and dutifulness than it is of daring and curiosity and persistence. You'll notice that the Four-Point Approach doesn't champion a lock-step procedure or a rigid set of rules for problem-solving success. Rather, it extols the benefits of exploration, experimentation, "trying stuff." Problems *are* problematic, that is their nature, but it is also what makes problem solving interesting and challenging and rewarding. So experiment freely with the Four-Point Approach and call out loudly for help or company on a regular basis!

SUMMARY CHART

POINT 1. **BREAK THE PROBLEM DOWN** Most problems do not have quick and obvious solutions. They involve a lot of information and many operations. Working towards a solution can best be accomplished by breaking the problem down. PARSE THE PROBLEM GENERATE RELEVANCIES LOCATE MODELS REPRESENT PROBLEMS SCHEMATICALLY ARTICULATE WHAT YOU DON'T KNOW	**POINT 2.** **WORK FOR AN UNDERSTANDING, NOT JUST AN ANSWER** Working just for the answers doesn't necessarily produce understanding, while working for understanding does tend to produce better problem-solving outcomes. TRY STUFF DOCUMENT YOUR THOUGHTS PAY SPECIAL ATTENTION TO ERRORS
POINT 3. **GIVE THE PROBLEM TIME** Problem solving takes time. Give yourself time for the false starts, confusions, and dead ends that problem solving inevitably requires. MONITOR YOUR TIME INCORPORATE YOUR PROBLEM SETS INTO YOUR STUDYING TAKE TIME TO *WORK* ON THE PROBLEM WITHOUT RUSHING TO *SOLVE* IT DON'T TREAT PROBLEMS AS THOUGH THEY WERE EXAMS	**POINT 4.** **DO YOUR SUMS!** **SUMMARIZE WHAT YOU HAVE LEARNED** Recognizing a concept or formula is not the same as being able to generate it from your own understanding or use it in solving novel problems. Practice summarizing your understandings in your own words. CLOSE THE BOOK THE MORAL OF THE STORY IS… TALK OUT LOUD HIT THE MEMORY HOT SPOT

QUOTATION SOURCES

p. 1 *Problems worthy of attack….*
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p. 2 *Don't give up…*
Layzer, D. (August 22, 1991). *Learning Mathematics.* Unpublished handout in Chemistry 8/9, Harvard University. (p. 4)

p. 3 *Good problem solvers…*
Whimby, A. and Lockhead, J. (1982). *Problem Solving and Comprehension*. The Franklin Institute Press. (p. 26)

p. 3 *A common misconception…*
Hallett, D. H. (1981). Understanding students who don't understand math. *New Directions for College Learning Assistance: Improving Mathematics Skills, No. 6*, pp. 3-7. Jossey Bass. (p. 5)

p. 5 *Let's try it…*
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p. 6 *Getting it wrong…*
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p. 7 *One must learn…*
Sophocles (1916). *Trachiniae.* In *The Dramas of Sophocles*, Dutton Press. (p. 191)

p. 8 *I would like to introduce…*
Wertime, R. (1979). Student problems and "courage spans." In J. Lockhead and J. Clement (Eds.). *Cognitive Process Instruction*, pp. 191-199. The Franklin Institute Press. (p. 196)

p. 10 *Only by trying…*
Tobias, S. (1987). *Succeed with Math.* The College Board. (p. 8)

p. 11 *A sudden "flash of insight"…*
Maddoxx, H. (1963). *How To Study.* Fawcett Publications. (p. 140)

"*The Four-Point Approach* is a tremendous resource for students looking to improve their problem solving skills ~ a must read! I enthusiastically recommend it."

– Thomas Torello
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