The Road To Digitopolis: Perils of Problem Solving

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How students solve problems is a topic of central concern both to educational researchers and to math/science teachers: What is the nature of good and poor problem solving? How can students improve their problem-solving capacities? Teachers are in a unique position to witness problem solving in action, and to draw connections between the classroom experiences of their students and the findings of research. This article presents an instance of problem solving (drawn from a popular children's book) annotated with references to current research in cognition and education. The annotations explore issues such as the effect of performance anxiety on problem solving, how problem solvers handle the experience of confusion, and the role of self-monitoring and metacognition in problem solving.

I recently ran across an inspired example of problem solving in an unexpected place: an old children's book, *The Phantom Tollbooth* (Juster, 1961, pp. 174-176). Near the end of the story, three characters grapple with a word problem — and make a messy job of it. Their efforts illustrate many of the psychological issues that are of central concern to today's math/science teachers and educational researchers:

What is involved in good vs. poor problem solving?

How do novice vs. expert problem solvers go about problem solving?

How do students learn to become better problem solvers?

The Tollbooth passage appears below, annotated with references to current research in cognition and education. The exercise is intended neither as literary criticism nor as a comprehensive research review. Rather, it is offered as an entertaining instance of what we do every day as teachers — try to draw useful connections between our observations of our students' struggles, on the one hand, and our understandings of the research literature, on the other.

For those unfamiliar with the Tollbooth story, some scene-setting is in order. The tale begins when a very bright but bored boy named Milo receives a mysterious package containing a map of "The Lands Beyond" and a kit for constructing a child-sized tollbooth. When Milo drives his toy car through the tollbooth, he finds himself on a strange adventure along with two traveling companions — a large Humbug and a friendly Watchdog named Tock. On the road to Digitopolis (the City of Numbers), they reach a junction and come to a stop, unsure of which way to go. When they ask directions of a strange creature with many faces, the Dodecahedron, he responds by posing them a problem.

"There's nothing to it. If a small car carrying three people at thirty miles an hour for ten minutes along a road five miles long at 11:35 in the morning starts at the same time as three people who have been traveling in a little automobile at twenty miles an hour for fifteen minutes on another road exactly twice as long as one half the distance of the other, while a dog, a bug, and a boy travel an equal distance in the same time or the same distance in an equal time along a third road in mid-October, then which one arrives first and which is the best way to go?"

"Seventeen!" shouted the Humbug, scribbling furiously on a piece of paper. [1]

"Well, I'm not sure, but—" Milo stammered after several minutes of frantic figuring. [2]

"You'll have to do better than that," scolded the Dodecahedron, "or you'll never know how far you've gone or whether or not you've ever gotten there." [3]

"I'm not very good at problems," admitted Milo. [4]

"What a shame," sighed the Dodecahedron, "They're so very useful. Why, did you know that if a beaver two feet long with a tail a foot and a half long can build a dam twelve feet high and six feet wide in two days, all you would need to build Boulder Dam is a beaver sixty-eight feet long with a fifty-one foot tail?" [5]

"Where would you find a beaver that big?" grumbled the Humbug as his pencil point snapped.

"I'm sure I don't know," he replied, "but if you did, you'd certainly know what to do with him."

"That's absurd," said Milo, whose head was spinning from all the numbers and questions.

"That may be true," he acknowledged, "but it's completely accurate, and as long as the answer is right,
who cares if the question is wrong? If you want to make sense, you'll have to make it yourself." [6]

"All three roads arrive at the same place at the same time," interrupted Tock, who had patiently been doing the first problem. [7]

"Correct!" shouted the Dodecahedron. "And I'll take you there myself. Now you can see how important problems are. If you hadn't done this one properly, you might have gone the wrong way." [8]

"I can't see where I made my mistake," said the Humbug, frantically rechecking his figures. [9]

"But if all the roads arrive at the same place at the same time, aren't they all the right way?" asked Milo.

"Certainly not!" he shouted, glaring from his most upset face. "They're all the wrong way. Just because you have a choice, it doesn't mean that any of them has to be right." [10] (Juster, 1961, pp. 174-176)

Annotations

[1] The Humbug feels pressured to produce an answer—quick! Deborah Hughes-Hallett, a professor of mathematics education at Harvard, has observed that inexperienced math students often try to answer a question the moment it is asked. She notes, "It is as though the student had been dropped by parachute in the middle of an unknown problem and, without waiting to identify what continent he was on and, therefore, what map to use, started to run for the finish line. Needless to say, without even having identified the problem, the student's chances of ending up in the right place are small indeed." (Hughes-Hallett, 1981, p. 6).

It is easy to spot a student who is using this approach, says Hughes-Hallett, because the student's answer generally "has little if anything to do with the original problem and, in fact, it often turns out that the original problem has been incorrectly identified" (Hughes-Hallett, 1981, p. 6). This certainly seems to describe the Humbug's shout of "Seventeen!" — - a strangely numerical answer given that the question was "Which is the best way to go?"! Answering the question asked, or more generally following the directions given, is critical to successful problem solving and test taking (BSC Study Strategy Materials, 1984; Tobias, 1987; Whimbey & Lockhead, 1982).

Whatever might have prompted the Humbug's quick answer, there are a number of situational and motivational variables that often lead students to jump the question:

a) Performance anxiety. When performance anxiety rises above an initially facilitative level, it has a deleterious effect on problem solving, making concentra-
answer, Milo expresses confusion (for which, unfortunately, he is about to be scolded by the Dodecahedron).

Confusion is a much maligned and misunderstood aspect of problem solving, largely because it is so frequently equated with failure. Sometimes confusion is interpreted as a sign that the student has failed at the task of learning; the student hasn’t been working hard enough or is just not smart enough. Sometimes confusion is interpreted as an indication that the teacher has failed at the task of teaching; the teacher didn’t explain clearly enough or organize the lesson well enough (Lipson, 1992).

But confusion is not necessarily a sign that anything has gone wrong, either with the teacher’s teaching or the student’s learning. In fact, much of what confuses students in their efforts to study science does so because it is inherently confusing (Lipson, 1990; 1992). The problem posed by the Dodecahedron is confusing mostly because of the length and complexity of the problem statement and the resulting cognitive overload for the reader.

John Clement and Clifford Konold of the Scientific Reasoning Research Institute at the University of Massachusetts/Amherst suggest that responding positively and productively to one’s own confusion is a critical problem-solving skill (Clement & Konold, 1989). Expert and novice problem solvers treat confusion differently. Experts greet their experiences of confusion as useful information which helps them monitor their progress and decide how to allocate their problem-solving energies most efficiently. Novices, on the other hand, greet their confusion as an interfering distraction or as a sign that they should just give up. Clement and Konold (1989) explain:

Whereas experts look for problem-specific causes of feelings of confusion, trying to reduce them by making adjustments to their solution process, novices give these feelings a more general interpretation, ignoring them if possible, or surrendering to them if not. The necessary process of engaging in a cycle of conjecture, evaluation, and correction requires both the attribution of confusion to problem-specific causes and breaking away from the common belief that problem solving always consists of recalling a well-defined procedure and executing it. (p. 29)

Students can learn, with the help of their teachers, to cope with confusion in a productive way. To manage confusion, the problem solver must explicitly acknowledge the confusion, locate and identify its sources, and design and implement strategies for resolving it. “Only when students are encouraged . . . to meet and engage their confusion — rather than deny it or run away from it in shame — will they begin to recognize confusion as a familiar and perhaps essential part of intellectual inquiry” (Lipson, 1992, p. 95).

[3] Given the above comments about confusion, it is unfortunate that the Dodecahedron treats Milo’s expression of uncertainty so harshly. It is precisely as a result of such harshness that students come to think of confusion as a shameful problem. The Dodecahedron goes on, however, to touch on a very important aspect of problem solving: the problem solver’s capacity for self-monitoring, for keeping track of “how far you’ve gone and whether or not you’ve ever gotten there.”

Research in metacognition and related fields has demonstrated how important it is for problem solvers to periodically pause in their work and ask themselves “Am I understanding this?” (Garner & Alexander, 1989, p. 145). Effective self-monitoring has several positive effects: it allows a problem solver to address comprehension difficulties or strategy failures as they occur; it reduces the number of repeated errors; and it contributes important information to time- and task-management decisions (see Garner, 1987).

In general, people tend to monitor themselves less than they optimally could and should, and their performance suffers as a result (Flavell, 1979; Pressley & Ghatala, 1990). Children and novices are especially poor at monitoring their progress and performance, whether the task at hand is reading expository text or solving a science problem. For example, in monitoring their reading comprehension, children tend to consistently overestimate their level of understanding and therefore are confident that their incorrect answers to comprehension questions are correct. (See Garner & Alexander, 1989, for a review of relevant research.) Similar effects are found with regard to problem solving (e.g., Chi, Bassok, Lewis, Reiman, & Glaser, 1989; Ferguson-Hessler & de Jong, 1990). Fortunately, self-monitoring capacities can be improved with training, as various programs of thinking-skills instruction have demonstrated (Nickerson, Perkins, & Smith, 1985, Perkins, 1992; Swartz & Perkins, 1989; ).

[4] Milo may or may not be very good at problems, but he thinks he is not, and this perception can play a critical role in task performance (Bandura, 1977; Brown, Bransford, Ferrara, & Campione, 1983; Lipson, 1981). It may be called an issue of self-esteem, or perceived control, or expectations for success—but if Milo doesn’t think he has the ability to solve the problem, he is less likely to exert the effort required to do so. This can
become a dangerously self-perpetuating cycle. He doesn't exert the necessary effort, so he fails the task. He concludes he is no good at it and has little hope of success, so he doesn't exert the effort, and fails again.

Our assessments of our capacities are often influenced by how we feel at the moment. Milo's confession that he's not good at problems may be based as much on his subjective and momentary sense of frustration as it is on any objective assessment of his prior or current performance. Novice problem solvers often, mistakenly, assume a correlation between their degree of felt confusion and their actual problem-solving progress — mistakenly, in that their confusion level might be high even though they are making good progress on the problem, or their confusion level might be low even though they are making poor progress (Lipson, 1995).

[5] The Dodecahedron's comment reminds us how important it is for a problem solver to determine what the problem (or problem-framer) is asking. If the Dodecahedron were to pause immediately after saying "all you would need to build Boulder Dam is" and wait for us to finish the sentence, we would no doubt try to frame an answer in units of beavers, not units of beaver-feet. But from the Dodecahedron's perspective, this effort would be a waste of time, producing an entirely inappropriate answer.

The importance of figuring out and keeping in mind what a problem is asking is a lesson that many students learn by experience and consider valuable. At Harvard, students who have done well in a science class can join a peer-tutoring program and assist other students in subsequent sessions of the course. When a group of the most experienced tutors were asked to state what words of wisdom they would pass along to their tutees, the advice "Always check your answer against the question!" was strongly endorsed. "Are you answering what the question asks? Is your answer in the right units? If the question asks for a temperature and your answer is a weight, you have gone astray somewhere!" (Lipson, 1994, p. 17).

[6] Notice that the Humbug and Milo both find the Dodecahedron's problem puzzling, but for quite different reasons. The Humbug accepts at face value the problem-world that the Dodecahedron defines; a frame of reference in which one computes how big a beaver you need to build Boulder Dam. The Humbug is puzzled only by a within-frame issue: where you would find a beaver that big. Milo, on the other hand, takes a step outside the problem-world defined by the Dodeca-

hedron, and responds to the inherent absurdity of solving the problem in units of beaver-feet rather than units of beavers. Milo and the Humbug are working on the basis of different epistemological assumptions; they stand differently in relationship to the Dodecahedron's "authority" to define the boundaries of the problem; they have different understandings regarding the contextuality of knowledge (Perry, 1970). (Note also that the Dodecahedron had to stand differently in relationship to the conventions of logic and bounds of reality to devise a problem about a huge beaver in the first place).

In many ways, Milo's view is the enlightened one, the preferred one. The ability to question and expand beyond a given frame of reference is valuable in many real-world problem-solving contexts and is often cited as a defining characteristic of creativity (Adams, 1974; de Bono, 1967; Koestler, 1964; Perkins, 1981; Stolzenberg, 1984). Yet in most traditional educational settings, tasks are narrowly defined and answers are strongly predetermined, so attempts to offer a frame-breaking solution are not rewarded (Duckworth, 1987). In fact, they are more likely to be considered uncooperative or smart-alecky. Imagine that the following problem is presented to students in a traditional classroom: "If a six-foot man can dig a 10 ft. trench in one hour, to dig a 20 ft. trench requires ..." It is the rare student who responds, "A 12 ft. man!" Yet the capacity for just this sort of creative frame-breaking lies at the heart of deep, rather than surface, inquiries into scientific or mathematic principles. The Dodecahedron asks, "as long as the answer is right, who cares ...?" Clearly, we would want our students to care deeply not only about the rightness of answers but also about the meaningfulness of the questions.

The Dodecahedron's final comment is perhaps most telling: "If you want sense, you'll have to make it yourself." Knowledge is not an attribute of so-called "objective" reality, he implies, but rather it is a fundamentally human construction. Meaning is made, and the job of the student or problem solver is to make it. The strongest statement of this notion appears in the philosophy of radical constructivism (von Glasersfeld, 1984; 1988; 1989; Watzlawick, 1984). The focus of current educational research on understanding the first-person experience of the learner (Maciuika, Basseches, & Lipson, 1994) is implicitly based in a constructivist philosophy. So is the tendency of the good teacher who, when confronted with a befuddled student, tries to determine the student's own construction of the problem and go on from there, rather than simply repeating for the student the "correct" answer.
that the cars start at different times and so arrive at the
which better serves the purpose and illustrates the point
into. In contrast, interpreting the problem to suggest
same time also disambiguates the problem, but in a way
possible numbers and recognize a formula to plug them
searching the problem only so far as to identify acces-
interpretation sidesteps the problem's ambiguities, by
rates and times stated in the problem, the cars will not
arrive at their destination at the same time. one could
argue alternatively that if the first two cars start at the
second car "have [already] been travelling" for a
time. The problem states both, so the problem solver
the second-year college physics class writes in her course
original text. We should bear this in mind when we
hear students' cries of "Where does it say that?"; "That's
not fair!"; "But it says right here . . . !"; or "Am I supposed
to be a mind-reader?" Students who are
thinking about a problem in very concrete terms
consider a teacher's reference to the purpose or prin-
ciple of the problem to be "pulled out of thin air," "beside
the point," or "just made up."

It's important to recognize that the first orientation
produces a "wrong" answer, not by any misguided leap
of interpretation, but by sticking too closely to the
original text. We should bear this in mind when we
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the point," or "just made up."

[9] Earlier, the Dodecahedron pointed out to Milo
the importance of self-monitoring. Here, the Humbug
is having difficulty monitoring his sources of error.
This is a particular problem in secondary and college
science training, and is often cited by students as an
area of great frustration in their problem solving. Not
only do they not understand, but they do not know what
how or why they do not understand. A student in a
first-year college physics class writes in her course
journal, "I don't know when to plug in what where,
when I try to do a physics problem which may (who
knows?) involve calculus." Another laments, "As
much as I tried to pull a problem apart, piece by piece,
it was like rebuilding a motor and having a few bolts
left over. What do I do? Where did this come from?
What do I do now?" (Lipson, 1992, p. 93).

Difficulties with error-detection are related to more
general difficulties with knowing what to do and when
to do it. A student may have studied the textbook
thoroughly and be able to pass a multiple-choice com-
prehension test with flying colors, and yet still not
know how to use the information or material in solving
a problem. Many students experience this distinction
when they watch a teacher solve a problem in class in
a way that looks completely obvious to them ("I knew
that!") even though they had sat stumped for hours by

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the very same problem. Their difficulties lie not with content knowledge—what they know—but with procedural knowledge—what they know how to do.

Alfred North Whitehead (1929) depicted this as a problem of “inert” knowledge (that is, knowledge that one can state but not employ), and identified such unusable knowledge as a common and undesirable result of poor educational practices. (See Bareiter & Scardamalia, 1985.) Related topics in recent research include, for example: metacognition (e.g., Flavell, 1976, 1979; Garner & Alexander, 1989; Wellman, 1983); the relationship between content knowledge, cognitive development, metacognition, and the use of cognitive strategies (e.g., Brown, Campione, & Day, 1981; Kohura-Kojima & Hatano, 1991; Prawat, 1991; Swanson, 1990); and self-regulation (e.g., Zimmerman & Schunk, 1989; Zimmerman, 1990).

[10] After expending all that problem-solving effort, Milo realizes that all roads lead to Digitopolis. Furthermore, he realizes that “Which is the right way to go among the three roads?”, “Which is the right way to go among all possible routes?”, and “Which roads lead to Digitopolis?” are not necessarily the same question. The Dodecahedron’s admonishment — just because one has a choice doesn’t mean any of them has to be right — is a reminder to Milo and the travelers to always keep a sharp eye on the big picture and on the criterion validity of their efforts. Problem solving is not an end in itself, but a means towards real-world understanding.

For many students, the problems they must solve in their math and science training are far removed from any real-world applications. Particularly in the introductory sciences at the college level, there seems to be an assumption that the student must simply survive information-dense, memorization-heavy survey courses as a right-of-passage and a weeding-out mechanism, before they get the chance at upper levels of study to do any “real” science. It is often not until late in their training that students have opportunities to learn, for example, that finding and framing problems are essential activities in the actual practice of scientific inquiry, where problem goals are “emergent” rather than pre-defined and cutting-edge problems do not come with an answer key.

Many high-ability and high-interest students drop out of science at this introductory level out of frustration with its limitations; not because they are failing, but because their science education is failing them (Lipson & Tobias, 1991; Tobias, 1990). A key characteristic of undergraduate science programs that work well (that have high retention and success rates) is that they involve students in hands-on research—“real” science—earlier in their training (Tobias, 1992).

Conclusion

Only a few of the psychological and educational issues which are reflected in the Tollbooth passage have been touched upon here. The passage is rich in this respect. It raises questions, evokes associations, invites musings.

But even an incomplete exercise in annotation confirms that a close look at the first-hand experience of problem solvers and an effort to bring that experience into conversation with the results of problem-solving research can be both entertaining and illuminating. Too often, problem solvers and scholars of problem solving go about their respective tasks in isolation from one another, without the benefit of shared conversation. And too often, we as teachers pass up opportunities to be the mediators of that conversation.

We can also take the Tollbooth passage as something of a morality tale, a warning to educators about what happens when problem solving is taught poorly or not at all. The results are terribly predictable and predictably terrible. Students become more concerned with getting it done than with getting it. They learn compliance rather than competence. They become masters at handling the sterile, piecemeal tasks set for them in school yet remain wholly unprepared for the realities of problem solving in natural contexts.

These are heavy lessons to draw from a light tale told for the entertainment of children. But we can learn important lessons wherever we find them. The Dodecahedron is right: If you want sense you just have to make it yourself.

References


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